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Similarly, $d=\frac{0}{m-n}$ means no distance from P, and shows that they were placed together by the conditions of the problem, $a=$zero, but for *all other time the problem is impossible.*

5. If $a=$zero, and $m=n$.

Then $t=\frac{0}{0}$, and $d=\frac{0}{0}$.

As any number of zeros subtracted from zero gives zero; that is, zero$-v\times 0=$zero, where v represents any number whatever, they are together at all times; for $t=$*any number*.

Hence, $t=\frac{0}{0}$ means all conceivable times, and $d=\frac{0}{0}$ means all conceivable distances, and are indeterminate, not being one, but every value.

Therefore, *A result* $\frac{0}{0}$ *indicates that the problem is indeterminate.*

The form, $\frac{0}{0}$, is a *symbol of indetermination*, and does not indicate that no solution can be found, but that too many can be determined. The indetermination consists in the fact that any one of the infinite solutions will answer just as well as any other.

January 11, 1897.

EDITORIALS.

This issue of the MONTHLY was delayed on account of securing sorts for Dr. Lovett's article.

Prof. E. L. Brown, formerly of the Capital University, Columbus, Ohio, is now a member of the Faculty of the Department of Mathematics of the Colorado State University.

The articles on "Euclidean Geometry Without Disputed Axioms," and "Zero, Infinitesimals, Infinity, and the Fundamental Symbol of Indetermination," are published at the request of the authors. They invite criticism on their respective articles, and, if there is any defect in the reasoning by which they arrived at their conclusions, they desire to have the same pointed out.

BOOKS AND PERIODICALS.

A Text-Book of Light. With Numerous Diagrams and Examples. By R. Wallace Stewart, D. Sc., London, Author of "An Elementary Text-Book of Heat and Light," "An Elementary Text-Book of Magnetism and Electricity," etc.

Third Edition. 8vo Cloth, 208 pages. Price, 3*s.* 6*d.* London : W. B. Cleve University Correspondence College Press. New York : Hinds & Noble, 4 Cooper Institute.

This little treatise on Light is clearly, neatly, and accurately written. The author as a teacher and writer needs no introduction. His works all bear evidence of a master of the subject under consideration. This book in the hands of the student will enable him to read with interest and profit the investigations in this most fascinating phenomenon of nature. B. F. F.

On the Transitive Substitution Groups that are Simply Isomorphic to the Symmetric or Alternating Group of Degree Six. By Dr. G. A. Miller.

This is a reprint of a paper read before the American Philosophical Society May 7, 1897, and published in the proceedings of that Society. B. F. F.

New Principles of Geometry with Complete Theory of Parallels. By Nicolái Ivánovish Lobachévski. Translated from the Russian by Dr. George Bruce Halsted. Volume fifth of the Neomonic Series.

The publication of the translation of this little pamphlet of 26 pages promised by Dr. Halsted at the Mathematical Congress of the World's Columbian Exposition was delayed for a personal visit to Kazan, the home of Lobachévski, and Maros Vásárhely, the home of Bolyai.

In this pamphlet is some interesting matter for the student of Non-Euclidean Geometry, and it should be read by every teacher of geometry. Several of our readers have written to us saying that they could see no sense in the Non-Euclidean Geometry. We say to these, read this little pamphlet, then with the light you get from it turn back and read the first and all subsequent articles of Dr. Halsted's which have appeared in the MONTHLY during the last four years, then turn to other sources for information on the subject. After having done this, you will find that there is a Non-Euclidean Geometry, that its argument is as rigorous as the Euclidean, and that its deductions are equally interesting. B. F. F.

A Text-Book of Physics. Largely Experimental, including the Harvard College "Descriptive List of Elementary Exercises in Physics" By Edwin H. Hall, Ph. D., Professor of Physics in Harvard College, and Joseph Y. Bergen, A. M., Instructor in the Harvard Summer School of Physics, and Junior Master in English High School, Boston. Revised and Enlarged. 8vo Cloth. 596 pages. New York : Henry Holt & Co.

This book needs no introduction to the public. The first edition has proved its usefulness in all experimental courses in Physics. The second edition is even an improvement over the first. B. F. F.

Ordinary Differential Equations. An Elementary Text-Book with an Introduction to Lie's Theory of the Group of One Parameter. By James Morris Page, Ph. D., University of Leipzig, Fellow by Courtesy Johns Hopkins University, Adjunct Professor of Pure Mathematics in University of Virginia. 8vo Cloth. 226 pages. New York and London : The Macmillan Co.

This is the best elementary exposition of the subject of Ordinary Differential Equations with which we are acquainted. It differs from the older text-books upon the subject in one important respect, namely, in the method of treatment. Instead of giving theories of integration for certain classes of Differential Equations, as for instance, the Homogeneous or Linear Differential Equations as is done in the older works on the subject, the author has followed the method of Professor Lie. In 1870, Lie showed that it is possible to subordinate all the older methods or theories of integration to a general method. By the method of Lie it is possible to derive all of the older theories from a common source and at the same time build a broader foundation for the general theory of Differential Equations. The simple, elegant, and clear presentation of the subject in this work makes it possible for a student who has ambition and a fair knowledge of Analytical Geometry and Calculus to master this book without an instructor. B. F. F.

On the Primitive Substitution Groups of Degree Fifteen. By Dr. G. A. Miller. Pamphlet, 12 pages.

This paper is an extract from the Proceedings of the London Mathematical Society, Vol. XXVIII., and is along Dr. Miller's favorite line of investigation. B. F. F.

Higher Arithmetic. By Wooster Woodruff Beman, Professor of Mathematics in the University of Michigan, and David Eugene Smith, Professor of Mathematics in the Michigan State Normal School. 12mo. Cloth and Leather Back. 194 pages. Price, 80 cents. Chicago : Ginn & Co.

Among the many valuable features of this work are the elimination of the traditional problems which have become the common property of nearly every arithmetic published during the last quarter century; the introduction, instead of the traditional problems, of simple problems arising in the study of elementary physics, as, for example, easy problems in Electrical Measurements, problems coming under the application of Boyle's Law, the law of Falling Bodies, Specific Gravity, etc.; the treatment of the Metric System in the first part of the book (page 59) and the frequent use made of it in the subsequent part; the introduction of the common graphic methods of representing statistics; and the complete omission of rules. The entire omission of rules is a very common feature of the arithmetics which are published at the present time and of those which have been published during the last three or four years. It is my belief that all rules that can not be established easily by the deductive method of reasoning should be set in *good print* in the arithmetics. This is especially the case with the rules in Mensuration. The student of arithmetic is in general not competent to follow the argument which establishes the rule for finding the area of a triangle when three sides are given. Yet it is better for the student that he commit this rule to memory, though he does not know how it is established, than to be ignorant of its existence and the means by which the area of triangles are computed when the sides are given. It seems to me that any arithmetic treating the subject of mensuration ought to give the rules for finding the surface and volume of the three round bodies, the area of parallelograms, circles, and triangles, the triangles having the base and altitude given or the three sides. To these might be added the rules for finding the surface and volume of prisms and pyramids. Aside from these I heartily believe in the omission of rules. The work before us does not omit the consideration of the most of the above geometrical magnitudes, but the rules are not expressly stated. This work of Professors Beman and Smith is, however, one that we can very cheerfully recommend. B. F. F.

A Brief Introduction to the Infinitesimal Calculus. Designed Especially to Aid in Reading Mathematical Economics and Statics. By Irving Fisher, Ph. D., Assistant Professor of Political Science in Yale University, Co-author of Phillips and Fisher's Elements of Geometry. 12mo. Cloth. 84 pages. Price, 75 cents. New York and London : The Macmillan Co.

This little work on the Calculus will be received with joy by a great army of students, teachers, and professors, who have lacked the time and courage to attack some of the more exhaustive works on the subject yet felt the need of a knowledge of the Calculus in order to enable them to read with intelligence the highest authorities on economic as well as other subjects. Dr. Fisher has prepared this little work with a special view of the needs of this class of students. Any one with a clear mind can very easily read and understand every sentence in this book. There is no metaphysical speculation nor obscure statements made in establishing its first principles.

In considering the formula $s=\frac{1}{2}gt^2$, where s=space a body falls under the influence of gravity in the time t, he says, pages 2 and 3, "Since the above formula holds true of *all* points, it holds true now, when the time is $t+\triangle t$ and the distance $s+\triangle s$. That is $s+\triangle s=16(t+\triangle t)^2$. This gives $s+\triangle s=16t^2+32t.\triangle t+16\triangle t^2$. But $s=16t^2$. Subtracting, we have

$$\triangle s=32t.\triangle t+16(\triangle t)^2,$$

$$\text{whence } \frac{\triangle s}{\triangle t}=32t+16\triangle t \text{ ....} (1).$$

This is the *average* velocity during the small interval $\triangle t$.

Thus, if $\triangle t=\frac{1}{2}$ second and t be five seconds, the average speed of the body during that half second (viz., the one beginning 5 seconds from the rest) is $32\times 5+19\times\frac{1}{2}$, or 168 per second. If we take $\frac{1}{160}$ of a second instead of $\frac{1}{2}$, we have $32\times 5+16\times\frac{1}{160}$, or 168.1 feet per second.

The speed at the very instant of completing the 5th second is obtained by putting $\triangle t=0$, which gives 160 as the instantaneous speed.

Now when $\triangle t=0$, we call it dt, because 0 would not remind us of the kind of quantity which vanished, whereas dt does suggest t, the magnitude which vanished. When $\triangle t$ becomes 0, or dt, $\triangle s$ evidently becomes zero too, for a body can not go any distance in no time. This zero we call ds. Equation (1) therefore becomes at the limit

$$\frac{ds}{dt}=32t+[16]dt,$$

$$\text{or } \frac{0}{0}=32t+0, \text{ which may be written}$$

$$\frac{ds}{dt}=32t \text{ ....} (2),$$

for we can neglect the zeros on the right, but not those on the left (the ratio of two zeros does not reduce to zero).

* * * * * * * * * * * * *

It may be objected to the reasoning in the last article that $\frac{ds}{dt}$ or $\frac{0}{0}$ is indeterminate. This is true. $\frac{0}{0}$ is equal to 2, or 19, or 1, or any number we we please. *But the limit* $\frac{\triangle s}{\triangle t}$ *is not indeterminate.* We thus use $\frac{ds}{dt}$ in two distinct senses, viz : $\lim\frac{\triangle s}{\triangle t}$ and $\frac{\lim\triangle s}{\lim\triangle t}$.

The first is determinate, the second is indeterminate, though for that very reason it may always be put equal to the first. Only the first, or $\lim\frac{\triangle s}{\triangle t}$ is important. This is the ultimate ratio of two vanishing quantities."

Excepting the statement in the next to the last sentence, viz., that $\lim\frac{\Delta s}{\Delta t}$ is determinate and $\frac{\lim\Delta s}{\lim\Delta t}$ indeterminate, we claim that Dr. Fisher has established the fundamental principles of the Differential Calculus in a simple, rigorous, and logical manner. By the principle of Limits, the $\lim\frac{\Delta s}{\Delta t}=\frac{\lim\Delta s}{\lim\Delta t}$, that is to say, the limit of the quotient of two variables equals the quotient of their limits. Then if one is determinate the other is determinate, or if one is indeterminate the other is indeterminate. They are, however, both determinate. The statement that dt is used instead of 0 to preserve the trace of the quantity that vanished will be considered by many mathematicians as the rankest sort of mathematical heresy, the reanimating of Berkeley's "ghost of departed quantities." But here too Dr. Fisher's position is absolutely impregnable, for, since $\frac{0}{0}$ is, *per se*, indeterminate, but determinate by the equation by which, in every case, it is defined, it may be replaced by the ratio of any two quantities which preserves the ratio that defines $\frac{0}{0}$. So in the case above, $\frac{0}{0}$ can be replaced by $\frac{ds}{dt}$ or $\frac{y}{x}$ or the quotient of any other two quantities which preserve the ratio $32t$. But $\frac{0}{0}$ is replaced by $\frac{ds}{dt}$ to preserve the trace of the quantities which vanished, and ds and dt can represent large or small parts of s and t. There are in general three possible ways by which the ratio $32t$ can be preserved by $\frac{ds}{dt}$. First, ds being considered a constant, and dt a variable; second, ds being a variable, and dt a constant; third, $\frac{ds}{dt}$ both being variables. Each of these three ways of viewing $\frac{ds}{dt}$ is used in the Calculus. This method of exposition is used in my classes with the result that students are enabled to use the Calculus, not as a machine by which to grind out problems, but as an instrument of research.

The American Monthly Review of Reviews. An International Illustrated Monthly Magazine. Edited by Dr. Albert Shaw. Price, $2.50 per year in advance. Single Number, 25 cents. The American Monthly Review of Reviews Co., 13 Astor Place, New York City.

The American Monthly Review of Reviews for October has several articles of unusual interest to women readers. Miss Frances Willard tells the story of the World's W. C. T. U. movement; Mrs. Ellen M. Henrotin, president of the General Federation of Women's Clubs, outlines the benefits of those organizations; Mrs. Sheldon Amos, of England, writes of a London Women's Club, and Miss Mary Taylor Blauvelt contributes an enlightening article on the opportunities for women at the English universities. B. F. F.

The Arena. An Illustrated Monthly Magazine. Edited by John Clark Ridpath, LL. D. Price, $2.50 per year in advance. Single Number, 25 cents. Boston: The Arena Co.

The Arena for October continues the battle for reform. The number is especially interesting and in some parts brilliant; it is in all parts aggressive and courageous. Hon. Charles A. Towne's article, "The New Ostracism," is in the author's best vein of critical analysis. In the course of the discussion he attacks with great vigor the plutocratic interference with professors in colleges and universities. Herman E. Taubeneck continues his cogent statistical attack on consecrated wealth. Judge Walter Clark sends out a powerful plea for the establishment of public rights over semi-public interests and institutions. The Editor of *The Arena* continues with unabated vigor his onslaught on the organized forces of plutocracy. His article, "Prosperity: the Sham and the Reality," is one of his strongest and best. Dr. Ridpath's exposition of the bottom purposes and methods of the money power is as caustic as it is true. Mary Platt Parmelee's article on the Political Philosophy of the Father of American Democracy is an original and forceful argument for popular liberties. B. O. Flower is again at his best pace in "The Latest Social Vision," in which he discusses the merits of Bellamy's "Equality." Perhaps the most radical and defiant article in the number is "The Dead Hand in the Church," by Rev. Clarence Lathbury, in which he attacks with destructive criticism the domination of the dead past over the living present in the church. "Hypnotism in its Scientific and Forensic Aspects" is the subject of an interesting and useful article by Marion L. Dawson. "Suicide: Is It Worth While?" is the caption of Charles B. Newcomb's startling study of one of the most interesting and painful themes of the age. The "Plaza of the Poets" is rich with the contributions of Ironquill, Junius Hempstead, Clinton Scollard, Reubie Carpenter, and Helena M. Richardson; while "The Editor's Evening" sparkles with its usual gems of social and poetical philosophy. Under "Book Reviews" the charming poems of Madison Cawein are set forth with merited commendation. B. F. F.

The Open Court. A Monthly Magazine Devoted to the Science of Religion, and the Religion of Science, and the Extension of the Religious Parliament Idea. Edited by Dr. Paul Carus. T. J. McCormack, Assistant Editor, and E. C. Hegeler and Mary Carus, Associate Editors. Price, $1.00 per year in advance. Single Copies, 10 cents. Chicago and London: The Open Court Publishing Co.

The following is the table of contents of the November number: "An Introduction to the Study of Ethnological Jurisprudence," by the Late Justice Albert Hermann Post, Bremen, Germany; "History of the People of Israel from the Beginning to the Destruction of Jerusalem by the Chaldeans," by C. H. Cornill, Professor of Theology in the University of Koenigsberg; "The Religion of Science; the Worship of Beneficence," by James Odgers Knutsford, England; "Death in Religious Art," by the Editor; "Vivisection from an Ethical Point of View: A Controversy," by Prof. Henry C. Mercer, and others; "Leonhard Euler," a biographical sketch by T. J. McCormack; "The Sacred Books of the Buddhists," by Albert J. Edmunds; "Brief Notes on some Recent French Philosophical Works;" Book Reviews, and Notes. Among the book reviews is a just estimate or criticism of "Finkel's Mathematical Solution Book;" the review contains about a page and a half, and is written by Assistant Editor T. J. McCormack. B. F. F.